Optimal Decision Making with Limited, Imperfect Information

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Decision Problems in Nature: Working with Unclear Data

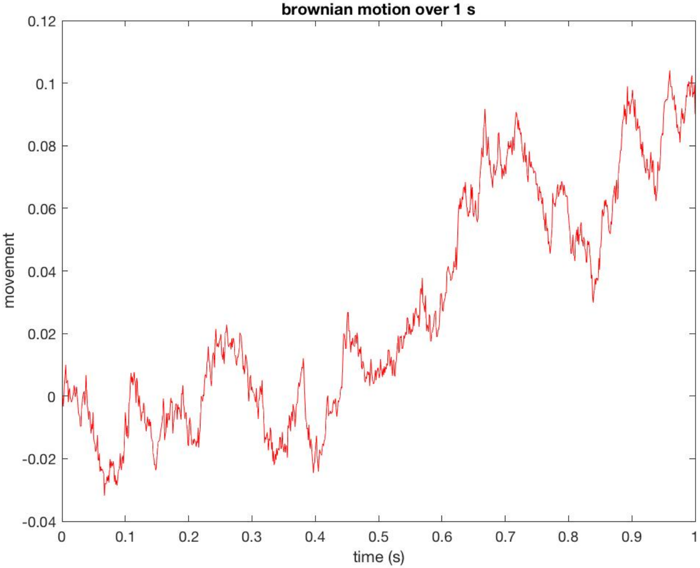
- Animals in the wild constantly have to make decisions to survive
- When they're safe, when something is edible, where to look for food
- Most important part of these decisions: they must be made with incomplete information, and must (sometimes) be made quickly
- Random, ambient changes both in the external environment and the animal's decision-making machinery must be accounted for

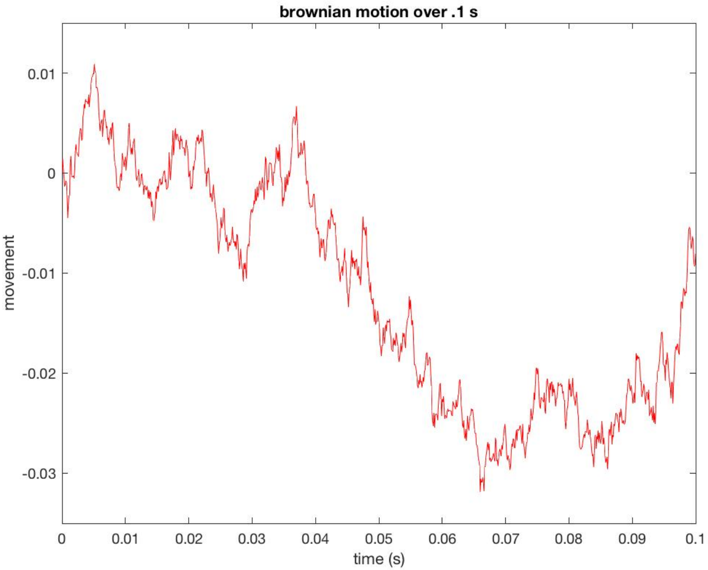
Goal:

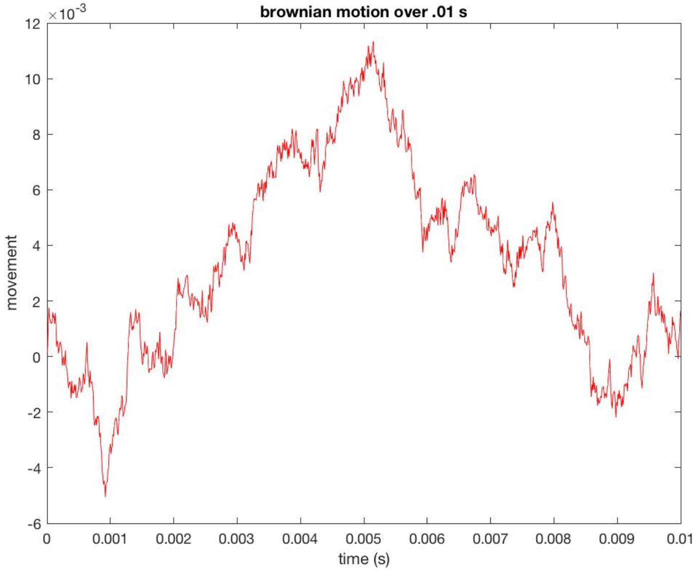
- "to examine which model or models can implement optimal decision-making, and use this to generate testable hypotheses about how social insects should behave if they are to decide optimally"
- Using stochastic differential equations to model the decision-making process.
- Taking inspiration from mathematical theory and neuron models to explain decision making in social insect colonies.

Modeling with Constant Random Change: Brownian Motion

- We're hoping to make a mathematical model for how decisions are made
- Need some way to account for constant, ambient changes in the evidence present
- Large concerns with scaling speed of decision-making, so rather than treating time as a set of discrete steps we must treat it continuously
- Brownian Motion is the simplest way of understanding continuous random changes mathematically

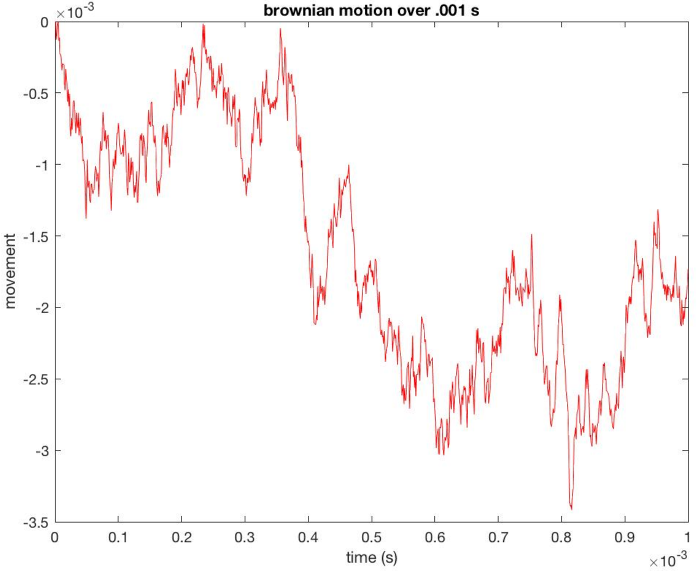

brownian motion over 1 s
movement
time (s)
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


brownian motion over .1 s
movement
time (s)
0.01
0
-0.01
-0.02
-0.03
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

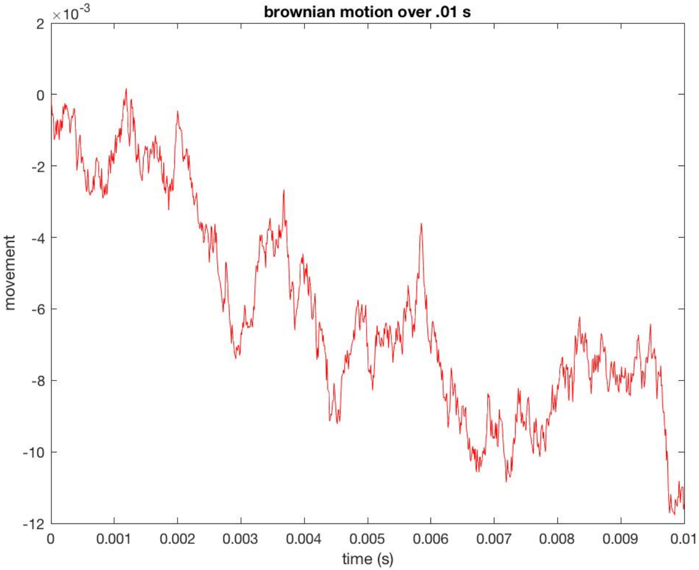

brownian motion over .01 s
×10^-3
12
10
8
6
4
2
0
-2
-4
-6
movement
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01
time (s)

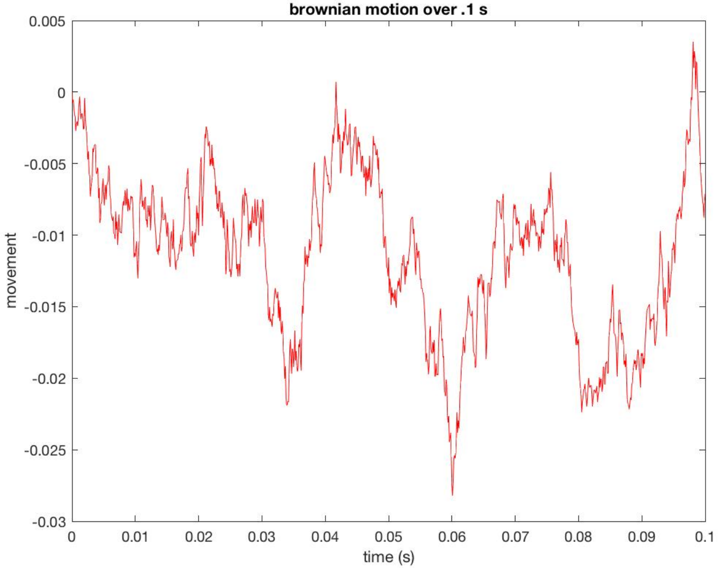
Choosing with a Noisy but Complete View: Biased Brownian Motion

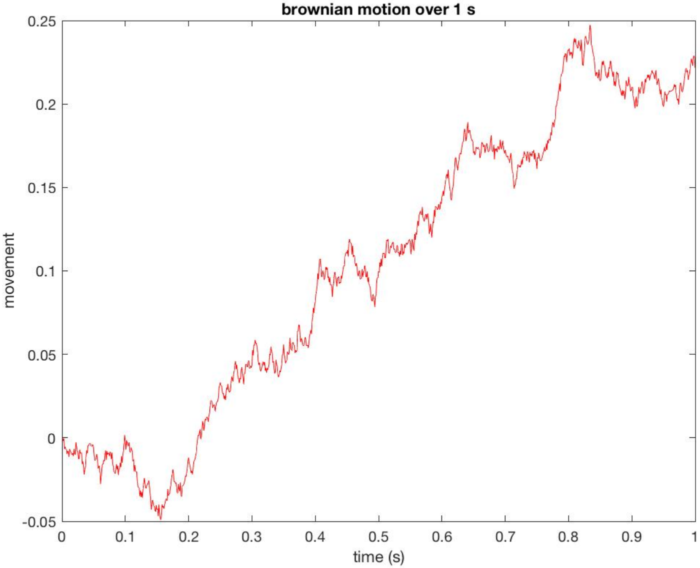
- When there's an unambiguous right answer (whether or not there's something hiding nearby), Brownian Motion doesn't tell the whole story.
- This is done by adding a bias to the motion. Movements regular Brownian Motion have a mean of 0, but we can change the mean to slightly more than 0
- The direction of the bias isn't always immediately apparent
- The best way to determine the direction of the bias is to set a threshold and wait until the process crosses that threshold.



brownian motion over .001 s
×10^-3
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
movement
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
time (s)
×10^-3


brownian motion over .01 s
×10^-3
2
0
-2
-4
-6
-8
-10
-12
movement
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01
time (s)


brownian motion over .1 s
movement
time (s)
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1


brownian motion over 1 s
movement
time (s)
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

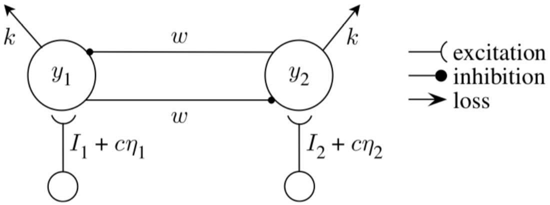
Biological Decision Making: A Simple Experiment

- To test decision making in primates, researchers showed primates a collection of moving dots.
- The primates had to determine whether the dots were mostly moving left or right, and look in the appropriate direction for a reward
- by varying the prizes based on how fast the primate guessed, researchers could vary the immediacy of the choice.
- https://m.youtube.com/watch?v=Cx5Ax68SIvk

Biological Decision Making: Experimental Results

- The primates trained with this experiment could vary their speed/certainty when given different reward structures
- Brain activity measurements showed that there were two areas that were activated in this experiment: medial temporal and lateral intraparietal
- A model was proposed to explain this behavior mathematically: Usher-McClelland

Optimal Neuron Firing: Usher-McClelland



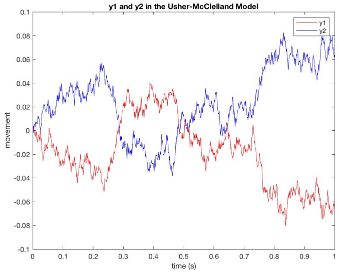
$$\begin{cases} \dot{y}_1 = I_1 + c\eta_1 - y_1 k - y_2 w, \\ \dot{y}_2 = I_2 + c\eta_2 - y_2 k - y_1 w, \end{cases}$$

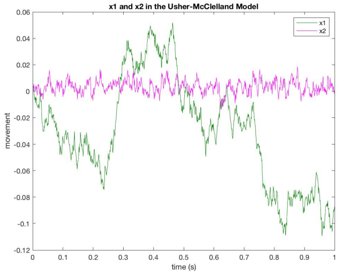
y_i is the charge in the neuron that makes choice i, k is the rate of forgetting, w is the extent to which mutually exclusive choices inhibit each other, I_i are the signals from the visual area in support of choice i, $c\eta_i$ is how much noise is present in I_i

Usher-McClelland Analysis

- The equations for Usher-McClelland are coupled (hard to work with) so we instead try to un-couple them.
- New equations can be given in terms of $x_1 \& x_2$, measuring the total support for either choice after taking both neurons into account and the disagreement between the neurons respectively.
- Findings were that if the inhibition and forgetting rate are the same (and both are high), the problem turns into a simple biased Brownian Motion problem, allowing the primates to tune the speed and accuracy of their responses.

Graphs of Usher-McClelland in action





Decision-Making in Social Insect Colonies

- Unanimous decision is required
- Highest quality site should be identified
- Quality-dependent recruitment
- Positive feedback
- Quorum Sensing

Finding a new Nest: 3 models, 2 species

T. albipennis (ant)

- Direct-switching model
- Recruiters use tandem running to teach others the route
- Recruiters pause longer before recruiting to poor nests than for good nests
- A decision is made when a site reaches a quorum amount of ants - the ants commit to that site and go back to nest and carry remaining members over

House-hunting in *T. Albipennis* (ant)

- Only modelling ants discovering nest sites and recruiting new members

$r_i'(s)$: rate at which recruiters recruit uncommitted scouts (s)

s : uncommitted scouting ants

$$r_i'(s) = \begin{cases} r_i' + c\eta_{r_i'} & s > 0 \\ 0 & \textit{otherwise} \end{cases}$$

House-hunting in *T. Albipennis* (ant)

y_i : recruiters for site i

q_i : rate at which uncommitted ants become recruiters

r_i : rate at which recruiters switch to recruiting for other site

k_i : rate at which recruiters switch to being uncommitted

$$\begin{cases} \dot{y_1} &= (n - y_1 - y_2)(q_1 + c\eta_{q_1}) + y_1 r_1'(s) \\ &+ y_2(r_2 + c\eta_{r_2}) - y_1(r_1 + c\eta_{r_1}) - y_1(k_1 + c\eta_{k_1}) \\ \dot{y_2} &= (n - y_1 - y_2)(q_2 + c\eta_{q_2}) + y_2 r_2'(s) \\ &+ y_1(r_1 + c\eta_{r_1}) - y_2(r_2 + c\eta_{r_2}) - y_2(k_2 + c\eta_{k_2}) \end{cases}$$

$$\begin{aligned} RecruitmentRateForSite_i &= Discovery + Recruitment \\ &+ SwitchingTo_i - SwitchingFrom_i \\ &- BecomingUncommitted \end{aligned}$$

Results:

- Would like to come up with random process $\dot{x}_1$, $\dot{x}_2$ that is identical to diffusion model
- Using the coordinate system from the User-McClelland model to decouple the differential equations
- The decay and switching rate parameters are dependent on qualities of both nest sites
- Optimal decision-making can only be achieved in this model if individuals have global knowledge about the alternatives available. (unrealistic)

House-hunting in *A. Mellifera*

- Ant model: direct-switching (not optimal)
- 1st Bee model: no direct-switching (not optimal)
- 2nd Bee model: direct-switching (optimal!)
  - Different from 1st ant model because the number of ants recruited over time is a linear function of the number of recruiters
  - Honeybees require both parties to meet, so the number of bees recruited per unit of time depends on the number of recruiters and also the number of uninformed recruits.
- Decision making in the second bee model becomes optimal when no uncommitted bees remain the colony.

Conclusion:

- Similarities were found between neural decision-making process, and collective decision-making process in social insect colonies.
- The direct switching bee model (A. mellifera) is the only model that plausibly approximates statistically optimal decision making.
- Hypothesis: Social insect colonies need to apply direct switching with recruitment to have an optimal decision making strategy.

Caveats:

- More research needs to be done to see if direct switching, or indirect switching is more biologically plausible.
- Conflating decision making with decision implementation (in ant model).
- Site discovery is a stochastic process - a good site might be discovered late in the process.
- The stochastic nature of site discovery is different from the neural model.
- Binary decision model is unlikely for insects searching for new nest site

Questions?